Chapter 1:
Introduction to Algebra I

Lesson 1: The Real Number System

Welcome to your first lesson in Algebra 1! When you think about topics associated with algebra, one of the topics that should come to mind is "numbers". Without numbers, algebra cannot exist. Today, you will explore the **real number system**.

The real number system contains all numbers as you know them. It includes non-decimals, decimals, fractions, positives, and negatives. But within the world of real numbers, there exists several more specific classifications of numbers.

Exercise #1: Fill in the definitions of each of the classifications of numbers in the real number system.

Rational Numbers:

____

____.

Irrational Numbers:

____

____.

Integers:

____

____.

Whole Numbers:

____

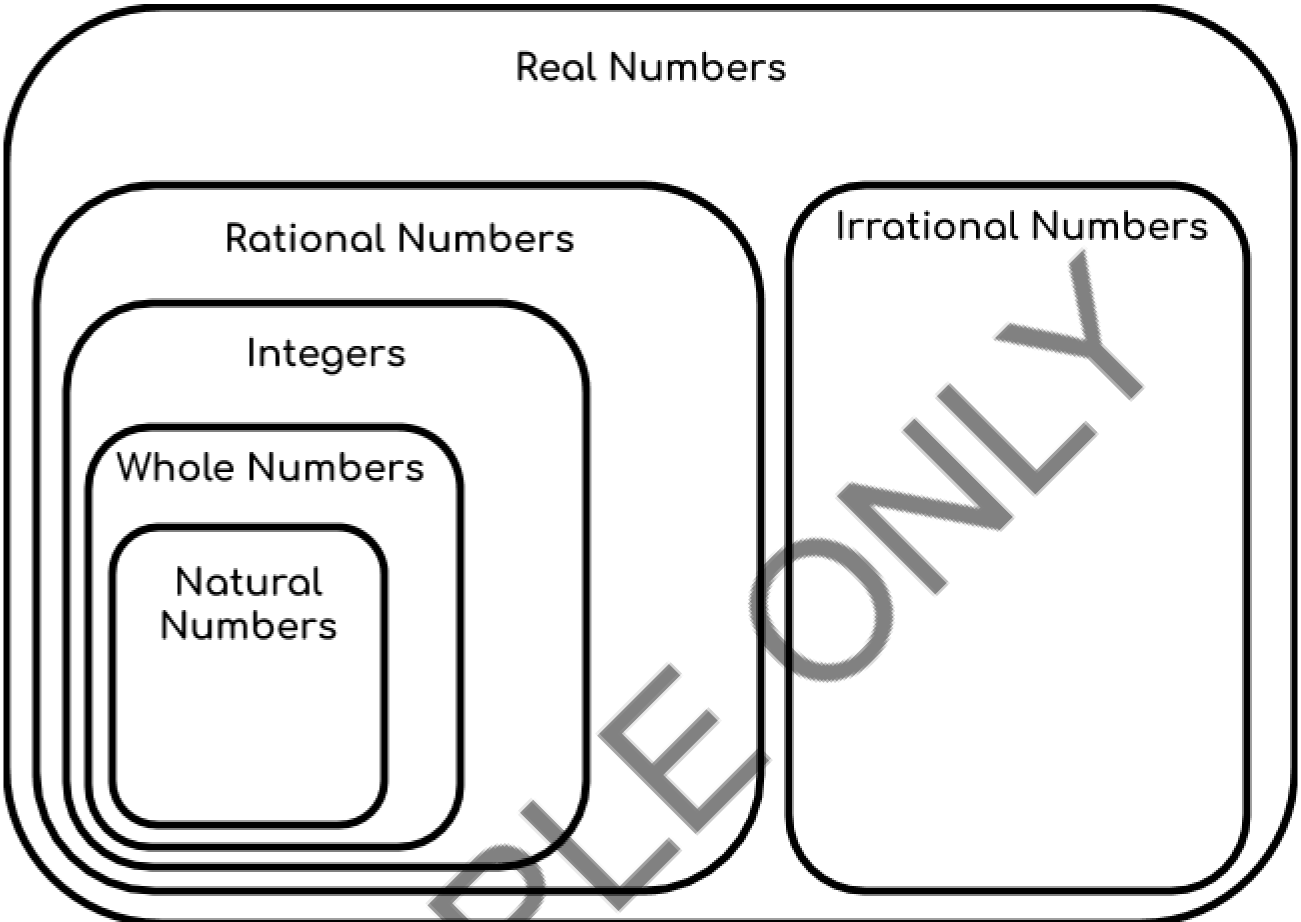
____.

Natural Numbers:

____

____.

Exercise #2: Think of three examples of each type of number and place them in the appropriate space in the diagram below.



Exercise #3: Determine if each of the following expressions are rational or irrational.

a) $\frac{1}{3} + \frac{3}{5}$

b) $\frac{1}{\sqrt{2}} + 8$

c) $(\frac{1}{\sqrt{5}})^2$

d) $\frac{1}{\sqrt{9}} + \frac{1}{\sqrt{25}}$

e) $\sqrt{8} \times 2$

f) 5π

Lesson 1 Extra Practice

EP1. Classify the following numbers as rational, irrational, integer, whole, and/or natural. Choose all that apply.

a) -5 ______________________________

b) $2,845$ ______________________________

c) $\sqrt{6.25}$ ______________________________

d) 0 ______________________________

e) $\sqrt{12}$ ______________________________

f) $\frac{4}{\sqrt{49}}$ ______________________________

EP2. Explain why the following numbers are either rational or irrational.

a) -0.667 ______________________________

b) $\frac{1}{9}$ ______________________________

c) $2.675675...$ ______________________________

d) $11.121314...$ ______________________________

e) $\frac{\pi}{2}$ ______________________________

EP3. Claire states that if a number is a perfect square, then the number must be even. Explain whether Claire is correct or incorrect.

EP4. State whether the result of each of the following is always rational, always irrational, or neither.

a) The product of a rational number and rational number.

b) The sum of an irrational number and an irrational number.

c) The sum of a rational number and an irrational number.

d) The product of an irrational number and an irrational number.

e) The product of a rational number and an irrational number.

f) The sum of a rational number and a rational number.

Lesson 2: Solving One Variable Linear Equations

Solving for an unknown variable is a key concept you will be building on throughout the year in Algebra. Today, you will master solving one variable linear equations.

Exercise #1: First begin by simplifying the expression below.

$$50 - 5(2 + 1)^2$$

Exercise #2: Explain how you went about simplifying the expression in Exercise #1. Is there a specific order you should follow to simplify expressions.

Exercise #3: When solving for the exact value of an unknown variable, it is important to follow this same order...in reverse.

a) Solve the equation below.

$$x - 8 = 9$$

b) Solve the equation below.

$$3x + 5 = 20$$

c) For each of the above equations, your intuition probably guided you to solve each equation. Can you see how the reverse order of operations was used?

Exercise #4: Using the reverse order of operations, solve each equation below for the given variable.

a) $2(x+5)=12$

b) $4-2(m-6)=18$

c) $\frac{1}{2}x-6=12$

d) $\frac{3}{4}p+6=\frac{1}{2}p-10$

Exercise #5: Now that you have practiced solving one variable linear equations, you will practice creating your own equations from clues given to you in the following consecutive integer riddles.

a) The sum of three consecutive integers is 66. What is the largest of these integers?

b) Find three consecutive odd integers such that the largest is one less than twice the smallest.

c) Find three consecutive integers such that when you add the smaller two integers, the result is five less than the largest integer.

Lesson 2 Extra Practice

EP1. Solve each equation below for the given variable.

a) $7(x+1)-6=134$

b) $4x+6=2(6-x)$

c) $\frac{3}{4}n+9=15$

d) $4+\frac{1}{4}(k-1)=7$

EP2. The lengths of the sides of a quadrilateral are consecutive odd integers. What is the length of the longest side if the perimeter is 120 units?

EP3. Find three consecutive integers whose sum is -54.

The problems below all involve a linear equation buried beneath the words. To solve each problem below, set up an appropriate equation to represent the scenario, then find the solution.

EP4. Bill and Colin are cousins. Right now, Bill is 21 years older than Colin. In 13 years, Bill's age will be twice Colin's age. How old is Colin right now? How old is Bill?

EP5. Three friends, Helga, Phoebe, and Gerald, found the sum of their ages to be 41. Helga is 3 years older than Phoebe. Gerald is 6 years less than twice Phoebe's age. How old is Phoebe?

EP6. For a sweet sixteen birthday party, Mr. and Mrs. Friedman are deciding between two different catering companies. Abe's Catering charges a $120 set up charge, plus $35 for each tray of food. Geri's Catering has no set up charge, and charges $50 for each tray of food. How many trays of food will Mr. and Mrs. Friedman have to order to be charged the same amount by each catering company?

Lesson 3: Rearranging Formulas

In the previous lesson, you worked with equations in one variable and isolated a variable to find out the exact value of that variable. In this lesson, you will be working with equations and formulas containing multiple variables, and rearranging them in order to isolate a specific variable.

Exercise #1: Consider the area formula for a quadrilateral, $A = bh$.

a) First, solve the following equation for h.

$$12 = 4h$$

b) Now, solve the formula for h, in terms of A and b, using the same operation.

$$A = bh$$

In both cases, you divide in order to isolate h. In part (b), however, unlike terms cannot be combined.

Exercise #2: Consider the formula to convert from degrees Fahrenheit to degrees Celsius, $C = \frac{5}{9}(F - 32)$.

a) First, solve the following equation for F.

$$100 = \frac{5}{9}(F - 32)$$

b) Now, solve the equation for F, in terms of C using the same operations

$$C = \frac{5}{9}(F - 32)$$

By isolating F, you have now created a formula to convert from degrees Celsius to degrees Fahrenheit.

Exercise #3: The following formulas are actual formulas used in mathematics, chemistry, physics, and/or biology. Solve for the specified variable.

a) Solve $d = \frac{m}{V}$ for m.

b) Solve $K = C + 273$ for C.

c) Solve $v_f = v_i + at$ for a.

d) Solve $d = v_i t + \frac{1}{2}at^2$ for v_i.

e) Solve $A = \frac{1}{2}h(b_1 + b_2)$ for b_1.

f) Solve $V = \frac{1}{3}\pi r^2 h$ for r.

Lesson 3 Extra Practice

EP1. Daniel, James, and Briella are given the following formula for perimeter of a rectangle. Each of them are asked to solve the formula for w. Their responses are noted below.

$$P = 2l + 2w$$

Daniel: $w = \frac{1}{2}p - l$ James: $w = \frac{1}{2}(p - 2l)$ Briella: $w = \frac{p-2l}{2}$

Who is correct? Justify your answer.

EP2. Colby invests his money and determines the amount of interest earned based on the formula $A = P + Prt$, where A represents the total amount of money, P represents the principal amount invested, r is the interest rate, and t is the time, in years.

a) Find the interest rate, r, in terms of A, P, and t.

b) Suppose Colby initially invested $4000, and two years later has $4280 with no additional deposits or withdrawals being made during that time period. Use the equation from part (a) to find the interest rate Colby is receiving on his investment.

EP3. Rearrange the following formulas to isolate the specified variable.

a) Solve $P = \frac{W}{t}$ for t ...then find t if $P = 80$ and $W = 320$.

b) Solve $A = 2\pi r^2 + 2\pi rh$ for h ...then find h if $A = 276.32$, $r = 4$, and $\pi = 3.14$.

c) Solve $M_1 V_1 = M_2 V_2$ for M_1 ...then find M_1 if $V_1 = 12$, $M_2 = 3$, and $V_2 = 8$.

d) Solve $m = \frac{Y_2 - Y_1}{X_2 - X_1}$ for Y_2 ...then find Y_2 if $m = \frac{2}{3}$, $X_1 = 12$, $X_2 = 27$, and $Y_1 = 14$.

Lesson 4: Solving One Variable Linear Inequalities

In Lesson 2, you practiced solving one variable linear equations. In this lesson you will practice solving one variable linear inequalities.

Exercise #1: Consider the inequality $1 > 0$.

a) What happens if you add 1 to both sides? Does it remain true?

b) What happens if you subtract 1 from both sides? Does it remain true?

c) What happens if you multiply both sides by 1? Does it remain true?

d) What happens if you divide both sides by 1? Does it remain true?

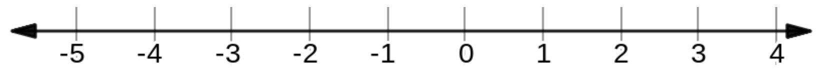
e) Can you think of <u>any</u> number you multiply both sides of the inequality by that would result in an inequality was not true?

f) Can you think of <u>any</u> number you could divide both sides of the inequality by that would result in an inequality that was not true?

g) Once you find the pattern in parts (e) and (f), state a simple rule that you could use in order to restore truth to the inequalities.

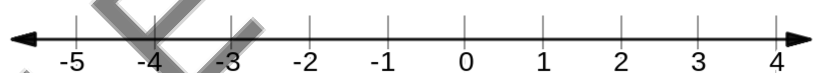
Exercise #2: Solve each of the inequalities below. Then, graph the solution set on the number line provided.

a) $12x + 2 < 14$

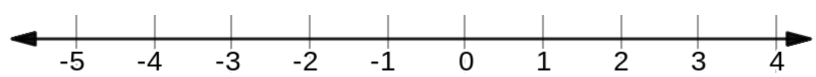


b) $-5x + 4 < -6$

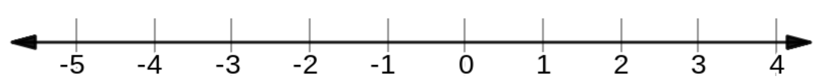
c) $8 \geq -3x - 1$



d) $4 - 2(x + 5) \leq -2$



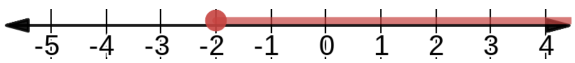
e) $5x + 7 > 4(6 - 3x)$



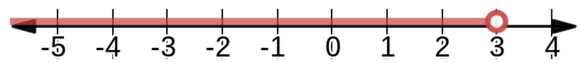
Lesson 4 Extra Practice

EP1. State the inequality represented by each graph below.

a) ______________________________

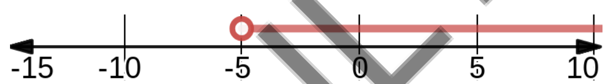


b) ______________________________



c) ______________________________

d) ______________________________



EP2. Consider the following solution set of an inequality.

Which of the following inequalities below represent the given solution set?

a) $8 - 2x > 32$

b) $-3(x - 6) < 27$

c) $\frac{1}{2}(x + 8) < 14$

d) $6 - \frac{3}{4}x < 15$

EP3. Solve each inequality below. Then, state the smallest possible integer that satisfies each inequality.

a) $3x - 9 \geq 30$

b) $-5(x-2) < 100$

c) $3(x+4) - 2(x+1) > 5$

d) $-8(x+2) - 9x + 2x \leq 14$

EP4. Consider the inequality below.

$$ax + 6 > b - 10$$

a) Solve the following inequality for x, given that $a < 0$.

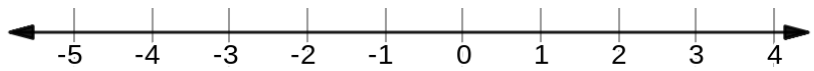
b) Suppose $a = -1$, and $b = 4$. What is the greatest possible integer that satisfies the inequality?

Lesson 5: Compound Inequalities and Interval Notation

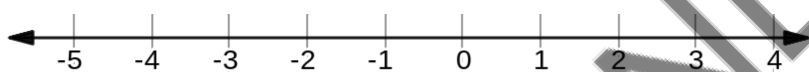
Words such as sunlight, classroom, and lifeguard, are called compound words. They are a combination of two different words into one. Today, you will learn about **compound inequalities**, or two separate inequalities combined into one.

Exercise #1: On the number lines below, graph each inequality.

a) $x \leq 3$



b) $x \geq -2$



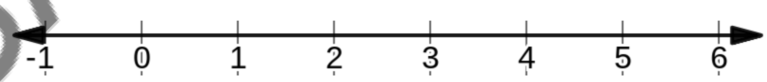
c) Where do the two inequalities overlap? The intersection of both solution sets could be written as a compound inequality. In the first blank space, write the smallest value both solution sets share, and in the second blank space, write the largest value both solution sets share.

$$\_\_\_\_ \leq x \leq \_\_\_\_$$

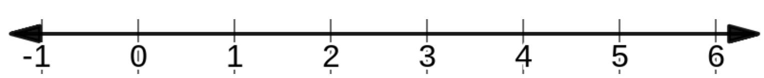
Exercise #2: Graph each compound inequality on the number line provided.

a) $0 \leq x \leq 4$

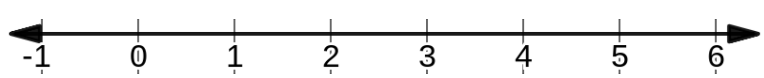
b) $0 \leq x < 4$



c) $2 < x \leq 5$



d) $1 < x < 6$

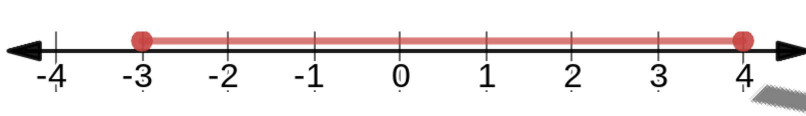


Now that you have practiced with some compound inequalities, you will learn how to write these compound inequalities in **interval notation**.

Interval notation is a shorthand way of writing compound inequalities. For closed circles, use either [or], and for open circles, use (or).

Exercise #3: Based on the solution sets graphed below, write each as a compound inequality and in interval notation.

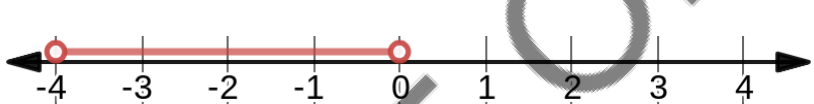
a)



Compound Inequality:

Interval Notation:

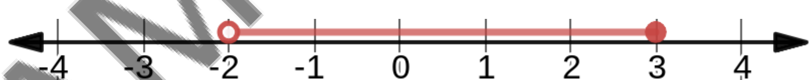
b)



Compound Inequality:

Interval Notation:

c)



Compound Inequality:

Interval Notation:

d)

Compound Inequality:

Interval Notation:

Lesson 5 Extra Practice

EP1. Graph each solution set based on the compound inequality or interval below.

a) $2 < x \leq 8$

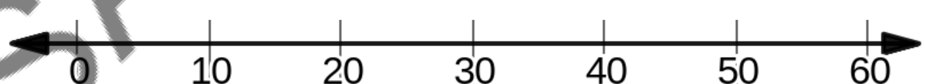
b) $[0, 12]$

c) $4 \leq x < 14$

d) $[6, 10)$

EP2. Graph the inequality represented by the interval below.

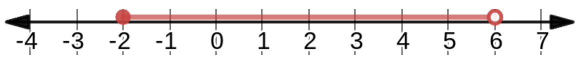
$$[20, \infty)$$



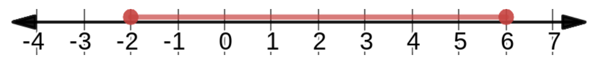
Explain why there is an open parenthesis,), with ∞ instead of a bracket,].

EP3. Which graph below represents the inequality $-2 < x \leq 6$?

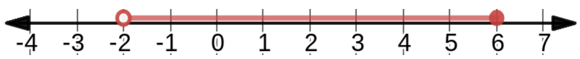
(1)



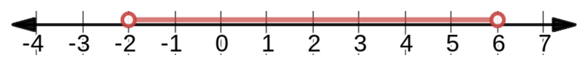
(3)



(2)



(4)



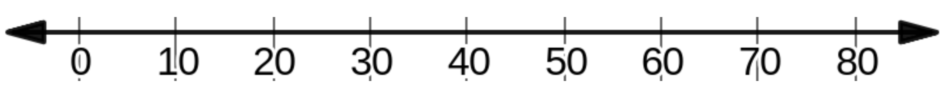
EP4. Thomas figures out that in order to make enough money at his job to help pay for college, he must work at least 10 hours per week. He also must work less than 20 hours per week, because of school and other obligations.

a) Graph the number of hours Thomas can work in one week on the number line below.

b) Write a compound inequality to represent your graph.

EP4. The temperature of the Atlantic Ocean was measured in Jones Beach, New York over the course of a year. The lowest temperature recorded was 38 degrees, and the highest temperature recorded was 76 degrees.

a) Graph the range of temperatures recorded on the number line below.



b) Write the range of temperatures recorded using interval notation.

Chapter Review

Part I Questions: For each statement or question, choose the word or expression that, of those given, best completes the statement or answers the question.

CR1. Given: $A = \sqrt{7}$
$B = 2\sqrt{3}$
$C = \sqrt{25}$
$D = \sqrt{1}$

Which expression results in a rational number?

1) $A + B$
2) $B + C$
3) $C + D$
4) $D + A$

CR2. The product of $\sqrt{1296}$ and $\sqrt{496}$ is

1) irrational because both factors are irrational
2) rational because both factors are rational
3) irrational because one factor is irrational
4) rational because one factor is rational

CR3. Given the following expressions:

I. $-\frac{5}{7} + \frac{2}{7}$ II. $\frac{1}{3} + \sqrt{11}$ III. $(\sqrt{2}) \cdot (\sqrt{2})$ IV. $5 \cdot (\sqrt{81})$

Which expression(s) result in an irrational number?

1) II, only
2) III, only
3) I, III, and IV
4) II, III, and IV

CR4. Which statement is *not* always true?

1) The product of two irrational numbers is irrational
2) The product of two rational numbers is rational
3) The sum of two rational numbers is rational
4) The sum of a rational number and an irrational number is irrational

CR5. Avogadro's Law provides the association between amount and volume of a gas. It can be represented by the formula $V_1n_1 = V_2n_2$. When the formula is solved for V_2, the result is

1) $V_1n_1n_2$
2) $\frac{n_2}{V_1n_1}$
3) $\frac{V_1n_1}{n_2}$
4) $\frac{V_1n_2}{n_1}$

CR6. The equation for the volume of a cone is $V = \frac{1}{3}\pi r^2h$. The positive value of r, in terms of h and V, is

1) $r = \sqrt{\frac{3V}{\pi h}}$
2) $r = \sqrt{3V\pi h}$
3) $r = \frac{V\pi h}{3}$
4) $r = \frac{V}{3\pi h}$

CR7. The solution to $4x - 9 = -2(5 - 3x)$ is

1) $\frac{1}{2}$
2) $\frac{1}{10}$
3) $-\frac{1}{2}$
4) $-\frac{1}{10}$

CR8. Which value of x satisfies the equation $\frac{29}{18} + \frac{1}{2}x = -\frac{5}{3}(x + \frac{1}{3})$?

1) $\frac{59}{39}$
2) $-\frac{59}{39}$
3) 1
4) -1

CR9. What is the value of x in the equation $\frac{x+6}{4} + \frac{1}{8} = \frac{7}{8}$?

1) -3
2) 0
3) 2
4) 3

CR10. What is the solution to $4k - 12 > 5k + 6$?

1) $k < 18$
2) $k < -18$
3) $k > 18$
4) $k > -18$

CR11. When $5x - 1 \leq 3(x + 4)$ is solved for x, the solution is

1) $x \leq \frac{13}{2}$
2) $x \geq \frac{13}{2}$
3) $x \leq 4$
4) $x \geq 4$

CR12. The inequality $5 - \frac{1}{3}x < x + 17$ is equivalent to

1) $x < -9$
2) $x > -9$
3) $x < -36$
4) $x > -36$

CR13. Which value would be a solution for x in the inequality $64 - 6x < 4$?

1) -10
2) 9
3) 10
4) 12

<u>Open Response Questions</u>: **For each question, clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc.**

CR14. Is the product of $\sqrt{36}$ and $\frac{1}{7}$ rational or irrational? Explain your reasoning.

CR15. Sophia decides that the sum of the expression $\frac{1}{9} + \frac{2\sqrt{13}}{7}$ is rational since it is a fraction. Is Sophia correct? Explain your reasoning.

CR16. Base your answer on the following numbers:

$a = \sqrt{45}$ $\qquad\qquad$ $b = 1.25$ $\qquad\qquad$ $c = \sqrt{324}$

Explain why $a + b$ is irrational, but $b + c$ is rational.

CR17. The formula for converting degrees Kelvin (K) to degrees Fahrenheit (F) is:

$$F = \frac{9}{5}K - 459.67$$

Solve for K, in terms of F.

CR18. Solve the equation below for the exact value of x.

$$8 - \tfrac{1}{5}(x + 2) = 2x$$

CR19. Solve the inequality below:

$$2.1 - 0.2p \geq 3.4 - 2.8p$$

CR20. Solve the inequality below to determine and state the largest possible value for x in the solution set.

$$7(x + 1) < 3x - 5$$

CR21. Solve for x algebraically:

$$5x - 4(2x + 10) \leq 2x + 12 - 9x$$

If x is a number in the interval $[10, 15]$, state all integers that satisfy the given inequality. Explain how you determined these values.

CR22. Given $3x + ax > -14$, determine the smallest integer value of a when $x = -2$.