Chapter 7: Polynomials

Lesson 1: Introduction to Polynomials

In this chapter, you will begin your study of polynomials. Polynomials are used in a variety of jobs, including but not limited to the fields of chemistry, physics, calculus, economics, and social sciences.

What is a monomial?

A monomial is a number, variable, or a product of a number and one or more variables with whole number exponents.

Exercise #1: What are examples of monomials? List as many as you would like below.

What is a polynomial?

A polynomial is a mathematical expression made up of two or more monomials.
A polynomial with two terms is called a **binomial**.
A polynomial with three terms is called a **trinomial**.

Exercise #2: For the following, identify if the polynomial is a monomial, binomial, trinomial, or none of the above.

a) $2x - 5$

b) $3x^2 + 5x + 2$

c) $4x^2 - 16$

d) 8

e) $4x + 3x$

f) $x^3 + 2x^2 - 4x + 7$

g) $x^2 + 6x + 9$

h) $4 \cdot 2x$

i) $x^4 - 81$

When you studied linear equations, you worked with $y = mx + b$. When you studied exponential growth and decay, you worked with $y = a(1 \pm r)^t$ When working with polynomials, you will be using the **standard form of a polynomial**.

A polynomial in one variable is in standard form when the exponents of the terms decrease from left to right.

Exercise #3: Place the following polynomials in standard form.

a) $3x + 2x^2 + 9x^4$ b) $4x^2 + 2x^3 - 7x + 12$ c) $7 - x^2 + 6x$

The **degree** of a polynomial is the largest exponent of the given variable, or the greatest degree of its terms.

Exercise #4: Place the following polynomials in standard form, then state the degree of the polynomial.

a) $5x - 2x^2 + 4$ b) $6x + x^3 - 5$ c) $4(x + 5) - 2x$

The **leading coefficient** of a polynomial is the coefficient of the term with the highest degree.

Exercise #5: State the leading coefficient of each polynomial.

a) $4x^2 + 2x - 16$ b) $6x - 3x^2 + 18$ c) $x^3 + 7x - 10$

Lesson 1 Extra Practice

EP1. For each of the following polynomials, circle the monomials, box the binomials, and underline the trinomials.

$5x$	$x^2 + 5x - 15$	$3x + 16$	$12 + 1$
$3x^2 + 7x$	$5(x + 1) - 5$	$2x^3 + 4x^2 - 8$	$16x - 12$

EP2. Match each of the polynomials on the left, with their equivalent polynomial in standard form on the right.

a)	$6x - 2x^2 - 7$	i)	$-2x^2 + 6x + 7$
b)	$2x^2 + 7 - 6x$	ii)	$-2x^2 + 6x - 7$
c)	$6x + 7 - 2x^2$	iii)	$-2x^2 - 6x - 7$
d)	$7 + 2x^2 + 6x$	iv)	$2x^2 - 6x + 7$
e)	$-2x^2 - 7 - 6x$	v)	$2x^2 + 6x + 7$

EP3. For each polynomial below, rewrite the expression in standard form, then state the degree of each polynomial.

a) $4x + 9x^2 - 3x^3 + 5$

b) $6 - x^4 + 2x^2 + 8x^3$

c) $8(2x + 3) - 16x$

d) $12 - 6(x + 2)$

EP4. For each polynomial below, rewrite the expression in standard form, then identify the leading coefficient.

a) $7x + x^2 - x^3$

b) $-5 + 3x^2 - 0.5x$

c) $-\frac{1}{20}x + 5x^2 - 16$

d) $\frac{1}{5}x^2 + 20 - \frac{3}{5}x$

Lesson 2: Adding and Subtracting Polynomials

Now that you have an understanding of the different parts of a polynomial and important terminology regarding polynomials, you will begin performing operations on polynomials.

In order to add or subtract polynomials, only terms with the **same variable** and **same exponent** can be combined.

Exercise #1: Add the polynomial expressions below. Express your answer as a polynomial in standard form.

a) $(3x^2 + 5x + 9) + (2x^2 - 4x - 3)$

b) $(2p + 8) + (8 - 6p)$

c) $(4 - x^2) + (x^2 + 10x - 16)$

d) $(8x - 3x^2 + 15) + (2x^2 + 12 - 14x)$

Exercise #2: Subtract the polynomial expressions below. Express your answer as a polynomial in standard form.

a) $(4x - 5) - (2x - 9)$

b) $(6x^2 + 15x - 9) - (x^2 + 10x - 8)$

c) $(18x + 24x^2 - 16) - (16x^2 - 22x + 54)$

d) $(\frac{1}{4}x^2 + \frac{1}{2}x - 1) - (\frac{3}{4}x^2 - \frac{1}{8}x)$

Exercise #3: Perform the indicated operation for each of the examples below. Express your answer as a polynomial in standard form.

a) Find the sum of $8x^2 + 5x - 6$ and $3x^2 - 16x + 19$

b) Subtract $x^2 - 7x + 13$ from $4x^2 - x + 8$

c) Find the difference of $-3x^2 + 7x + 17$ and $2x - 16$.

d) Add $x^3 + 2x^2 - 7x + 12$ and $4x^3 - 6x^2 + 14x - 1$

Exercise #4: A triangle has side lengths represented by polynomial expressions. If the side lengths of the triangle are represented by $2x^2 + 6$, $5x - 8$, and $x^2 + 6x - 20$, what is the perimeter of the triangle?

Exercise #5: A company represents its revenue by the function $R(x) = 6x^2 + 13x - 22$ and its costs by the function $C(x) = x^2 - 2x - 10$. If the company finds its profit using the function $P(x) = R(x) - C(x)$, express the profit of this company as a polynomial expression in standard form.

Lesson 2 Extra Practice

EP1. Perform the indicated operations for each exercise. Express your answer as a polynomial in standard form.

a) $(3x^2 + 2x - 9) + (6x^2 - 17x + 30)$

b) $(14x^2 - 19x + 5) + (x^2 + 12x - 8)$

c) $2(x^2 + 6x + 14) + (3x^2 - 2x + 15)$

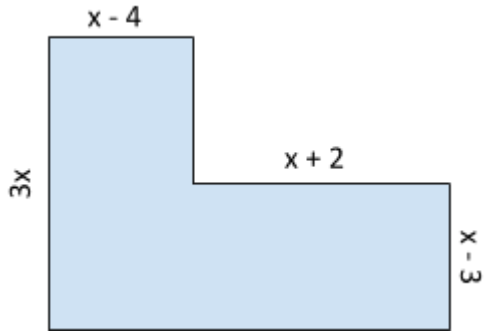
d) $(5x^2 - 18x - 20) - (7x^2 + 14x - 12)$

e) $(3x + 6) - 4(2x^2 + 7x - 12)$

f) $4(x^2 + 5x - 18) - 2(x^3 + 3x^2 - 6)$

EP2. The length of a rectangle is expressed as $4x^2 + x - 6$ and the width is expressed as $6x - 3$. What is the perimeter of the rectangle?

EP3. The irregular polygon below has side lengths expressed in terms of x. Find the perimeter of the irregular polygon below. Express your answer as a binomial.



EP4. Express the polynomial below as a binomial in standard form.

$$2 \cdot (4x^2 + 12x - 8) - 5(x^2 - 6x + 18) + 9(2x^2 - 6x + 12)$$

Lesson 3: Multiplying Polynomials

In the previous lesson, you learned how to add and subtract polynomial expressions. In this lesson, you will learn how to multiply a monomial by a polynomial. Your exponent rules from Chapter 5 will be very useful when multiplying a monomial by a polynomial.

Exercise #1: In each of the following examples, multiply the monomial by the polynomial.

a) $3x\,(6x - 7)$

b) $7x\,(x^2 + 8)$

c) $x^2(5x - 9)$

d) $9x\,(x^2 + 8x - 10)$

e) $10x^2\,(x^3 - 5x^2 + 13x)$

f) $0.2x^4\,(8x^5 + 20x^3 - 1x)$

Exercise #2: The length of a rectangle is expressed as $15x^2$ and the width of the rectangle is expressed as $2x^2 + 3x - 1$. What is the area of the rectangle, in terms of x?

Exercise #3: The base of a triangle is expressed as $8x^4$ and the height is expressed as $2x^3 - 6x$. What is the area of the triangle, in terms of x?

Lesson 3 Extra Practice

EP1. Multiply each of the following. Express your answer as a polynomial in standard form.

a) $(x^3)(6x^2 + 8x - 12)$

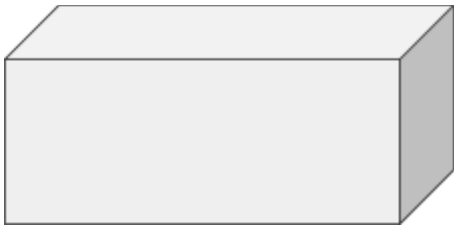
b) $(4x^2)(5x^6 + 3x^3 - 2x + 4)$

c) $\frac{1}{4}x^3(28x^4 - 12x^3 - 2x^2 + 80x - 36)$

d) $-6x^2(12x^2 - 7x - 3)$

EP2. Suppose x is an integer. Express the product of twice the integer and four more than three times the integer as a binomial in standard form.

EP3. Write a polynomial expression in standard form to represent the volume of the figure below if the length is represented by x^3, the width is represented by $\frac{1}{2}x^2$, and the height is represented by $8x - 12$.



Lesson 4: More Multiplying Polynomials

Now that you have practiced multiplying a monomial by a polynomial, you are ready to begin multiplying various polynomials by polynomials.

Let's say you want to send two letters in the mail. On each letter you must place a stamp and an address label. You would place a stamp on the first letter, then the second letter, and then you would place an address label on the first letter, then the second letter. Now, each letter has a stamp and an address label.

If we transfer this idea over to Algebra for finding the product of two polynomial expressions, we use a very similar process.

Consider the expression on the right.	$(x+8)(x+2)$
First, distribute the first term of the first set of parentheses to every term in the second set of parentheses.	$x(x+2)$
Then, distribute the second term of the first set of parentheses to every term in the second set of parentheses.	$8(x+2)$
Essentially, you are "double distributing" when you multiply a binomial by a polynomial.	$(x+8)(x+2)$ $x(x+2)+8(x+2)$ $x^2+2x+8x+16$
Simplify the expression by combining like terms, if necessary.	$x^2+10x+16$

The terms in the first set of parentheses could be seen as the stamp and the address label mentioned in the introduction. The terms in the second set of parentheses could be seen as the two letters that were to be mailed. Just as each letter must have a stamp and an address label, every term in the first set of parentheses must be multiplied by every term in the second set of parentheses.

Exercise #1: Simplify. Express your answer as a polynomial in standard form.

$(x+5)(x+9)$

Exercise #2: Simplify. Express your answer as a polynomial in standard form.

$(x+2)(x-6)$

Exercise #3: For each of the following, multiply the expressions by using double distribution.

a) $(x+7)(x-9)$ b) $(x-2)(x+11)$ c) $(x-1)(x-8)$

d) $(2x+1)^2$ e) $(x+4)(5x-3)$ f) $(x^2+8)(x+2)$

Exercise #4: Multiply the following expressions.

a) $(x+9)(x-9)$ b) $(x+4)(x-4)$ c) $(3x-2)(3x+2)$

d) Do you notice a pattern with the products of parts (a) - (c)?

Exercise #5: Use your knowledge of distribution to multiply the following polynomial expressions.

a) $(x+4)(x^2+5x+6)$ b) $(x^2-12)(x^2+6x-16)$

Exercise #6: Camille has a rectangular painting. The length of the painting is three times the width. She wants to put a frame around the painting that will make it 4 inches longer and 4 inches wider. Express the area of the painting with the frame as a polynomial in standard form.

Exercise #7: The edge of a cube is represented by the expression $2x - 3$. What is the volume of the cube? Express your answer as a polynomial in standard form.

Exercise #8: Express the product of three consecutive integers as a polynomial in standard form when the smallest of the three integers is m.

Exercise #9: Using your knowledge of multiplying polynomials, simplify the expression below.

$$(x^2 + 5x - 6)(x^2 - 5x - 24)$$

Lesson 4 Extra Practice

EP1. Multiply each of the polynomial expressions below.

a) $(x-3)(x+7)$ b) $(2x+9)(x-1)$ c) $4(x-2)(x+5)$

d) $(5x-11)(2x-7)$ e) $(x+3)^3$ f) $(3x^2-10)(x-4)$

EP2. The length of Jonathan's room can be expressed as $5x-2$ and the width of his room can be expressed as $2x+9$. Write an expression to represent the area of Jonathan's room.

EP3. The radius of a circle is expressed as $x+2$. What is area of the circle, in terms of x and π.

Chapter Review

Part I Questions: **For each statement or question, choose the word or expression that, of those given, best completes the statement or answers the question.**

CR1. An expression of the sixth degree is written with a leading coefficient of five and a constant of four. Which expression is correctly written for these conditions?

1) $6x^5 + x^4 + 7$
2) $5x^6 - 6x^4 + 5$
3) $6x^5 - x^5 + 5$
4) $5x^6 + 2x^2 + 4$

CR2. Mrs. Erickson asked her students to identify which of the polynomials below are in standard form and explain why.

I. $4x^3 + 5x^2 - 7x$
II. $10x^4 + 8x^2 + 2x$
III. $8x^6 - 3x + 2x^3 - 1$

Which student's response is correct?

1) Elias said II and III because the coefficients are decreasing.
2) Maya said only II because all the numbers are decreasing.
3) Griffen said I and II because the exponents are decreasing.
4) Summer said I and II because they each have three terms.

CR3. Which polynomial is twice the sum of $5x^2 - 3x + 4$ and $-7x^2 + 2x - 6$?

1) $-2x^2 - x - 2$
2) $-2x^2 - 5x - 10$
3) $-4x^2 - 2x - 4$
4) $-4x^2 - 10x - 20$

CR4. If $y = 3x^2 + x - 5$ and $z = x^2 - 6x + 12$, which polynomial is equivalent to $(y - z)$?

1) $2x^2 - 5x + 7$
2) $2x^2 + 7x - 17$
3) $-2x^2 - 7x + 17$
4) $4x^2 - 5x + 7$

CR5. Which trinomial is equivalent to $4(x - 2)^2 - 3(x - 1)$?

1) $4x^2 - 19x + 19$
2) $4x^2 - 13x + 15$
3) $4x^2 - 19x + 13$
4) $16x^2 - 3x - 61$

CR6. When $(x-5)^2$ is subtracted from $2x^2$ the result is

1) $x^2-10x+25$
2) $x^2+10x-25$
3) x^2+25
4) $-x^2-25$

CR7. The length, width, and height of a rectangular box are represented by $\frac{1}{2}x$, $2x+4$, and $4x-6$, respectively. When the volume is expressed as a polynomial in standard form, what is the coefficient of the 2nd term?

1) 14
2) 8
3) 4
4) 2

CR8. What is the perimeter of a regular hexagon with a side whose length is $2x+3$?

1) $12x+18$
2) $4x^2+12x+9$
3) $8x+12$
4) $4x^2+9$

<u>Open Response Questions</u>: **For each question, clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc.**

CR9. Subtract $6x^2+5x-14$ from $4x^2-8x-12$. Express the result as a trinomial.

CR10. Express the product of $3x^2+5x-9$ and $x-3$ in standard form.

CR11. Write the expression $3x+3x^2(6x+2)-5x^2-4x$ as a polynomial in standard form.