Chapter 8: Factoring

Lesson 1: GCF Factoring

In Chapter 7, you were introduced to polynomials and you spent some time multiplying different kinds of polynomials. You were given two (or more) polynomials, and by multiplying the polynomials, you created one polynomial in standard form.

In this chapter, you will begin with polynomials in standard form and you will "split" the polynomial into factors of simpler expressions. Essentially, you will be working backwards with what you learned in the previous chapter. In Algebra, this process is called **factoring**, and it is one of the most important topics of the Algebra curriculum.

Exercise #1: Think about the common factors of each of the terms of the binomial $6x^3 + 2x^2$.

a) What is the **Greatest Common Factor** of the coefficients of the terms?

b) What is the Greatest Common Factor of the variables of the terms?

c) Using parts (a) and (b), express the Greatest Common Factor (GCF) of the binomial.

d) After the GCF is **factored out** of the original expression, what terms remain?

e) Express the factorization of as a product of the GCF and the remaining terms.

Exercise #2: Consider the following expression: $4x^2 - 8x + 12$.

a) What is the **Greatest Common Factor** (GCF) of the expression?

b) Factor out the GCF.

Exercise #3: For each of the polynomials below, factor out the Greatest Common Factor. Your final answer should be expressed as the product of the GCF and the remaining terms.

a) $14x^2 + 21x - 7$

b) $18x^2 - 12x + 9$

c) $12x^4 + 10x^3 + 20x^2$

d) $45x^2 - 25$

e) $27x^4 - 54x^3$

f) $28x^4 - 40x^3 + 8x$

Exercise #4: Factor out the GCF from the polynomial expression below. Express your factorization as a product of the GCF and the remaining terms.

$$72m^5n^3 + 81m^3n^5 - 36m^2n^2 + 108mn^4$$

Lesson 1 Extra Practice

EP1. Factor out the Greatest Common Factor (GCF) for each of the expressions below.

a) $2x^2 + 4x + 16$

b) $13x^2 + 52x - 26$

c) $22x^3 - 55x^2 + 121x$

d) $16x^2 + 12x - 32$

e) $42x^5 + 84x^4 - 18x^3$

f) $24x^2 - 48x - 96$

g) $-5x^2 + 35x - 60$

h) $40x^2 - 90$

EP2. The area of a rectangle is represented by the expression $4x + 12$. If the length of the rectangle is 4, what is the width?

EP3. The area of a rectangle is represented by the expression $3x^2 - 12x$. If the width of the rectangle is $3x$, what is the length?

EP4. The area of a rectangle is represented by the expression $8x^3 + 4x^2$. What are possible dimensions of this rectangle, in terms of x?

EP5. Factor the GCF out of the polynomial expression below.

$$50x^3y^5 + 75x^2y^3 - 100xy^6$$

Lesson 2: Factoring Trinomials (Simple)

In this lesson you will be factoring trinomials that may not have a GCF. Multiplication fluency will be very useful when factoring trinomials.

Exercise #1: Let's begin by deriving the rules for factoring a simple polynomial.

a) Multiply the expression $(x+2)(x+5)$. Express your answer as a trinomial in standard form.

b) Multiply the expression $(x+4)(x+3)$. Express your answer as a trinomial in standard form.

c) Do you see a relationship between the original expressions and the final solutions? Describe a pattern between the original expressions, and the middle term and constant term.

In Algebra, the coefficients of the x^2 term, the x term, and the constant, are referred to as the a term, the b term, and the c term, respectively. So, the standard form of a trinomial could be expressed as:

$$ax^2 + bx + c$$

This will be very important to know going forth into the Algebra curriculum.

Exercise #2: Factor the following trinomials by finding two numbers that multiply to the c term, and combine to the b term.

a) $x^2 + 13x + 40$ b) $x^2 + 4x + 3$ c) $x^2 + 10x + 24$

Exercise #3: The numbers in factored form will not always be two positive numbers. Knowing that there could be negative numbers as well, factor the following trinomials.

a) $x^2 + 8x - 20$ b) $x^2 + x - 30$ c) $x^2 - 6x + 8$

Exercise #4: Before you factor each trinomial, first determine if you are able to factor out a Greatest Common Factor.

a) $2x^2 + 26x - 60$ b) $3x^2 - 3x - 126$ c) $5x^3 - 15x^2 - 140x$

Exercise #5: Here is a mix of all types of trinomial factoring you have covered so far. Factor each of the trinomials completely.

a) $x^2 + 20x + 100$ b) $x^2 + 17x + 72$ c) $x^2 - 5x - 36$

d) $4x^2 + 20x - 24$ e) $x^2 - 12x + 36$ f) $4x^2 + 16x - 128$

Lesson 2 Extra Practice

EP1. Factor the trinomials below.

a) $x^2 - 11x + 18$

b) $x^2 - 2x - 48$

c) $x^2 + 3x - 28$

d) $x^2 + 13x + 30$

e) $x^2 - 2x + 1$

f) $x^2 - 8x - 20$

EP2. Factor the trinomials below. Always look for a GCF as your first step in factoring!

a) $2x^2 + 16x - 96$

b) $3x^2 - 24x - 99$

c) $\frac{1}{2}x^2 - 2x - 30$

d) $5x^2 + 25x + 20$

EP3. A rectangular room has an area represented by $x^2 + 14x - 32$. If the width of the room is expressed as $(x + 16)$, what is the length of the room, in terms of x?

EP4. Robert is attempting to factor the trinomial $x^2 + 24x + 80$. He thinks the factors are $(x - 20)$ and $(x - 4)$. Is Robert correct or incorrect? Justify your answer.

EP5. The following polynomials follow the same rules for factoring as before, but look a little different. Try to factor each polynomial using your knowledge of factoring.

a) $x^2 - 16$

b) $x^2 + 13x$

c) $3x^2 + 42x$

d) $5x^2 - 180$

Lesson 3: Factoring a Difference of Perfect Squares & Perfect Square Trinomials

Now that you know how to factor simple trinomials, you will work with some polynomials that have unique properties.

Exercise #1: Multiply the polynomial expressions below.

a) $(x+2)(x-2)$ b) $(x-7)(x+7)$ c) $(x+10)(x-10)$

d) What do you notice about each of the multiplied expressions?

Exercise #2: Using the pattern you found in Exercise #1, factor the following polynomials.

a) x^2-1 b) x^2-9 c) x^2-81

d) x^2-64 e) x^2-400 f) x^2-144

Exercise #3: Now, try to factor the following polynomials. If it is not able to be factored, write "not factorable". Always look for a GCF as your first step in factoring!

a) $5x^2 - 20$ b) $2x^2 + 72$ c) $10x^2 - 90$

Exercise #4: Finally, given the following polynomials, factor *completely.*

a) $x^4 - 81$ b) $7x^2 - 28$ c) $16x^4 - 625$

In addition to factoring a difference of perfect squares, you will also factor perfect square trinomials. A perfect square trinomial is a trinomial that, when factored, has two factors that are exactly the same expression.

Exercise #5: Factor the perfect square trinomial expressions below. Write the factorization in the form $(x+h)^2$.

a) $x^2 + 12x + 36$ b) $x^2 + 4x + 4$ c) $x^2 - 8x + 16$

d) $x^2 - 14x + 49$ e) $x^2 + 16x + 64$ f) $x^4 + 10x^2 + 25$

Lesson 3 Extra Practice

EP1. Match each polynomial expression on the left with the correct factorization on the right.

_____ a.	$4x^2 + 40x + 100$	i.	$(x + 4)(x - 4)$
_____ b.	$x^2 + 6x + 9$	ii.	$(x + 3)^2$
_____ c.	$x^2 - 16$	iii.	$(x - 4)^2$
_____ d.	$x^2 - 9$	iv.	$4(x + 5)^2$
_____ e.	$4x^2 - 100$	v.	$(x - 3)(x + 3)$
_____ f.	$x^2 - 8x + 16$	vi.	$4(x + 5)(x - 5)$

EP2. Factor the following polynomials completely.

a) $x^3 - 4x$ b) $2x^5 - 32x$ c) $3x^4 + 75$

d) $x^2 + 100$ e) $9x^2 - 100$ f) $16x^2 - 25$

EP3. Here is a mixed review of all types of polynomial factoring you have learned thus far in Chapter 8. Factor each of the expressions completely.

a) $x^2 + 24x + 144$

b) $2x^2 - 60x + 450$

c) $4x^2 - 144$

d) $2x^2 + 162$

e) $x^2 - 4x - 32$

f) $2x^2 - 72$

g) $x^2 + 6x + 9$

h) $3x^2 - 21x - 90$

i) $4x^2 - 80x + 400$

j) $32x^4 - 162$

Lesson 4: Factoring Trinomials (Advanced)

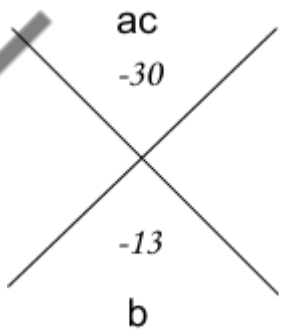
In this lesson you will practice factoring trinomials where the a term is not equal to 1. In Lessons 3 and 4, you worked mostly with trinomials that had an a term equal to 1, or a greatest common factor (GCF). If a trinomial does not have a GCF, this will change how you factor the trinomial.

Exercise #1: Consider the trinomial below.

$$3x^2 - 13x - 10$$

There are many different ways to factor trinomials when the a term is not equal to 1, and there is no GCF. This lesson will focus on the $a \cdot c$ method.

Step 1: Use the figure to the right, and the trinomial above, to fill in the product of $a \cdot c$ in the top portion of the figure, and the b term in the bottom portion of the figure.



Step 2: Think of two numbers that will multiply to the $a \cdot c$ term, and combine to result in the b term. Write them on the left and right side of the figure.

Step 3: Now that you have your two numbers, place them in the parentheses below. Notice, instead of x, the first term is now $3x$.

$$(3x \quad)(3x \quad)$$

Step 4: Finally, divide the a term out of one set of parentheses. You must do this because in the beginning of the problem we multiplied $a \cdot c$ and created an entirely new number. In order to get back to the original c term of -10, you must divide the $a \cdot c$ term by a.

$$(3x + 2)(x - 5)$$

If you multiply this expression using distribution, what is the result?

Exercise #2: Factor the polynomials below using the method on the previous page.

a) $2x^2 + 3x - 9$

b) $5x^2 + 19x + 12$

c) $3x^2 - 10x + 7$

d) $7x^2 + 6x - 1$

Exercise #3: If neither set of parentheses are divisible by the a term, you may need to divide *each* set of parentheses by a factor of the a term. Factor each trinomial below.

a) $9x^2 + 9x - 40$

b) $6x^2 + 7x - 49$

Lesson 4 Extra Practice

EP1. Factor each of the trinomials below.

a) $3x^2 - 2x - 5$

b) $2x^2 + 11x + 5$

c) $7x^2 - 11x + 4$

d) $5x^2 + 17x + 6$

e) $3x^2 + 10x - 8$

f) $2x^2 + 13x + 15$

EP2. Factor each of the trinomials below.

a) $4x^2 + 8x + 3$

b) $8x^2 - 10x + 3$

c) $15x^2 - 28x - 32$

d) $12x^2 + 17x + 6$

EP3. Jakub and Hayden are attempting to factor the polynomial $x + 2x^2 - 15$. Jakub says the polynomial could be factored to $(x + 5)(x - 3)$, since $-3 \cdot 5 = -15$ and $-3 + 5 = 2$, and Hayden says the polynomial is not factorable. Explain if either of them is correct, and justify your answer completely.

Lesson 5: Choosing the Best Method to Factor

Each lesson in this unit is organized by a different method of factoring. However, in this lesson, you must decide which of the factoring methods you have learned thus far would be the best way to factor each polynomial.

Exercise #1: Factor each of the polynomials completely.

a) $200x^2 - 50$

b) $7x^3 + 14x^2 + 7x$

c) $2x^2 - x - 3$

d) $4x^2 - 20x + 25$

e) $4x^2 - 49$

f) $12x^2 - 44x + 7$

g) $2x^2 + 20x + 48$

h) $3x^2 - 11x - 20$

i) $2x^2 - 36x + 162$

Exercise #2: Complete the puzzle below by factoring each polynomial. Once you have factored the polynomial, find the matching factorization in the answer bank, then, place the word in the corresponding box to decipher the hidden message.

A. $x^2 + 3x - 4$
B. $3x^2 + 8x + 5$
C. $2x^2 - x - 6$
D. $6x^2 + 5x - 4$
E. $x^2 - 12x + 36$
F. $x^3 + 2x^2 + x$
G. $3x^3 + 2x^2 - x$
H. $2x^2 - 34x - 36$
I. $3x^2 - 24x - 60$

Did You Hear About…?

A.	B.	C.
D.	E.	F.
G.	H.	I.

Answer Bank:

$(2x + 3)(x - 2)$ AT	$2(x - 18)(x + 1)$ IN	$2(x + 18)(x - 1)$ ELEPHANT
$x(x + 1)(3x - 1)$ WAS	$x(x + 1)^2$ IT	$(3x + 5)(x + 1)$ FIRE
$3(x - 10)(x + 2)$ TENTS	$x(x - 1)(3x + 1)$ BURN	$(3x + 4)(2x - 1)$ THE
$(x + 6)^2$ BARN?	$(x + 4)(x - 1)$ THE	$(x - 6)^2$ CIRCUS?

Lesson 5 Extra Practice

EP1. Match each polynomial on the left with its equivalent factorization on the right.

_____ a.	$x^2 - 16x + 63$	i.	$(7x + 4)(x - 5)$
_____ b.	$7x^2 - 31x - 20$	ii.	$3x(x + 3)$
_____ c.	$3x^2 + 9x$	iii.	$2x(x + 1)(x - 1)$
_____ d.	$x^2 - 5x - 14$	iv.	$(x - 9)(x - 7)$
_____ e.	$2x^2 + 17x + 21$	v.	$(2x + 3)(x + 7)$
_____ f.	$2x^3 - 2x$	vi.	$(x + 2)(x - 7)$

EP2. For what values of b is the expression below factorable?

$$x^2 + bx + 12$$

EP3. List four values of c that make the expression below factorable.

$$x^2 - 3x + c$$

EP4. The area of a rectangle is expressed as $6x^2 + 7x + 2$ square inches and the length is expressed as $(3x + 2)$ inches. What is the width of the rectangle?

EP5. A square has an area expressed as $25x^2 + 20x + 4$ square feet. Find the length of the square.

Chapter Review

Part I Questions: For each statement or question, choose the word or expression that, of those given, best completes the statement or answers the question.

CR1. Elaina correctly factored the expression $p^2 - 5p - 36$. Which expression did she write?

1) $(p-6)(p+6)$
2) $(p-6)(p-6)$
3) $(p+4)(p-9)$
4) $(p+9)(p-4)$

CR2. The trinomial $x^2 - 18x + 81$ can be expressed as

1) $(x+9)^2$
2) $(x-9)^2$
3) $(x-9)(x+9)$
4) $(x+1)(x+81)$

CR3. Four expressions are shown below.

I. $6(x^2 + 2x - 8)$
II. $6(x-2)(x+4)$
III. $3(2x+8)(x-2)$
IV. $2x(3x+6) - 48$

The expression $6x^2 + 12x - 48$ is equivalent to

1) I and II, only
2) II and IV, only
3) I, II, and III
4) I, II, III, and IV

CR4. When factored completely, $x^3 - 13x^2 + 30x$ is

1) $x(x+3)(x-10)$
2) $x(x-3)(x-10)$
3) $x(x+2)(x-15)$
4) $x(x-2)(x+15)$

CR5. Which expression is equivalent to $x^4 - 16x^2 + 64$?

1) $(x^2+8)(x^2-8)$
2) $(x^2+8)(x^2+8)$
3) $(x^2-8)(x^2-8)$
4) $(x^2+8)(x+4)(x-2)$

CR6. The expression $25x^2 - 49$ is equivalent to

1) $(5x - 7)^2$
2) $(12.5x - 24.5)^2$
3) $(5x + 7)(5x - 7)$
4) $(12.5x - 24.5)(12.5x + 24.5)$

CR7. Which expression is equivalent to $36x^2 - 144$?

1) $36(x - 2)(x - 2)$
2) $4(3x - 6)(3x - 6)$
3) $2(9x - 72)(9x + 72)$
4) $3(2x + 4)(6x - 12)$

CR8. The expression $x^4 - 1$ is equivalent to

1) $(x^2 + 1)(x^2 - 1)$
2) $(x^2 - 1)(x^2 - 1)$
3) $(x^2 + 1)(x - 1)(x - 1)$
4) $(x^2 - 1)(x^2 - 1)$

CR9. When factored completely, the expression $p^4 - 16$ is equivalent to

1) $(p + 2)(p - 2)(p + 2)(p - 2)$
2) $(p^2 + 4)(p + 2)(p - 2)$
3) $(p^2 - 4)(p^2 - 4)$
4) $(p^2 + 4)(p^2 - 4)$

<u>Open Response Questions</u>: **For each question, clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc.**

CR10. In the equation $x^2 - 3x + 28 = (x + a)(x + b)$, b is an integer. Find algebraically *all* possible values of b.

CR11. Factor the expression $x^4 - 2x^2 - 8$ completely.